Introduction to Reinforcement Learning

Lecture 4: POMDPs & Multi-Agent RL

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(based on material from Frans Oliehoek, Tony Cassandra, Michael Littman, and Leslie Kaelbling)

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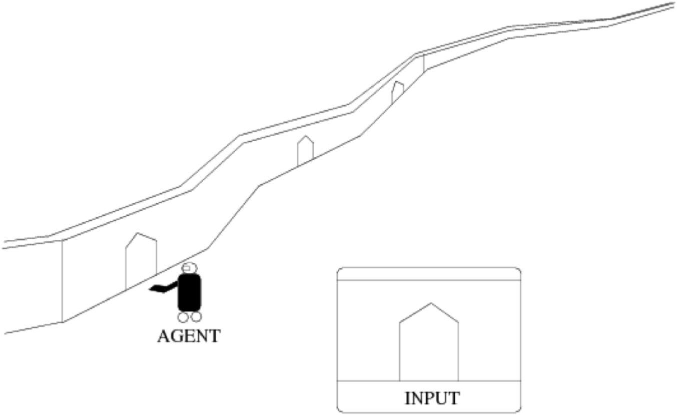
Partial observability

- In a *partially observable* decision problem, the agent does not have access to the true state of the environment
- Instead agent receives only *observations* correlated with the state
- There are two possible causes of partial observability:
  1. Noisy sensors: many-to-many function mapping states to observations
  2. Perceptual aliasing: many-to-one mapping

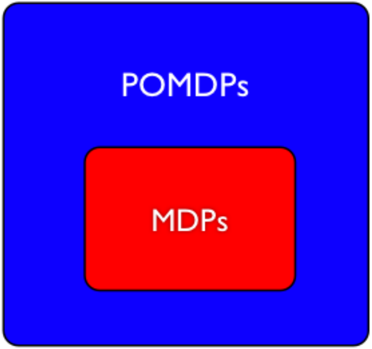
Hallway example

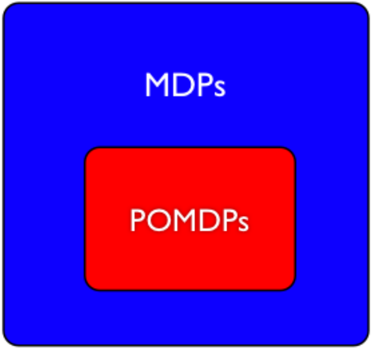


Partially observable Markov decision processes

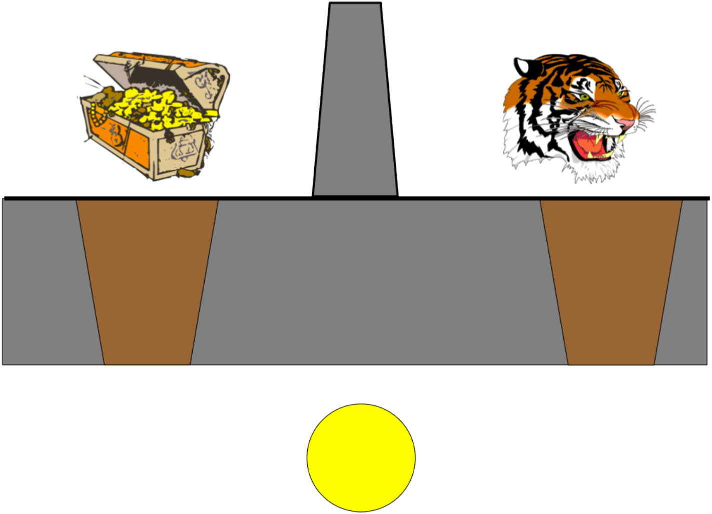
- POMDPs extend MDPs to model partial observability
- Environment is stationary and possibly stochastic environment
- A finite POMDP consists of:
  - Discrete time $t = 0, 1, 2, \dots$
  - A discrete set of states $s \in S$
  - A discrete set of observations $o \in O$
  - A discrete set of actions $a \in A$
  - A *transition model* $p(s'|s,a)$: the probability of transitioning to state s' when the agent takes action a at state s
  - An *observation model* $p(o|s',a)$: the probability of receiving an observation o after taking action a and landing in state s'
  - A *reward* function $R : S \times A \mapsto \mathbb{R}$, so that the agent receives reward $R(s,a)$ when it takes action a at state s
  - A planning horizon, which can be infinite

MDPs $\subset$ POMDPs or POMDPs $\subset$ MDPs?





Tiger example (1)



Tiger example (2)

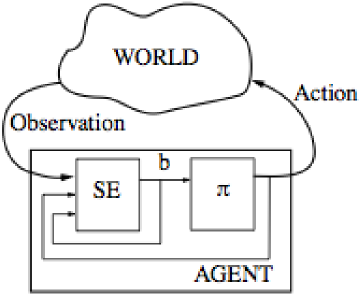
- $S = \{L, R\}$
- $A = \{OL, OR, Li\}$
- $O = \{HL, HR\}$
- Transitions: state is static but opening resets
- Rewards:
    - Correct door: $+10$
    - Wrong door: -100
    - Listen: -1
- Observations: correct 85% of the time:
    - $p(HL|L, Li) = 0.85$
    - $p(HR|L, Li) = 0.15$
    - $p(HL|R, Li) = 0.15$
    - $p(HR|R, Li) = 0.85$

Heuristic approaches

- Without Markov property, reactive policies are suboptimal
- Can sometimes settle for them anyway
- Or condition actions on:
    - Entire history
    - A fixed window of history
    - An engineered subset of history
    - An engineered higher-level observation

Beliefs (1)

- Principled approaches formalise uncertainty about the state
- A *belief* is a probability distribution over states, conditioned on what the agent has observed: $b(s) = p(s)$



Beliefs (2)

- Updating the belief requires knowledge of b, T, and O
- Start from Bayes rule:

$$p(A|B) = \frac{p(A,B)}{p(B)} = \frac{p(B|A)p(A)}{p(B)}$$

- In our case:

$$p(s'|o) = \frac{p(o|s')p(s')}{p(o)}$$

- Adding other givens:

$$p(s'|o,a,b) = \frac{p(o|s',a,b)p(s'|a,b)}{p(o|a,b)}$$

Beliefs (3)

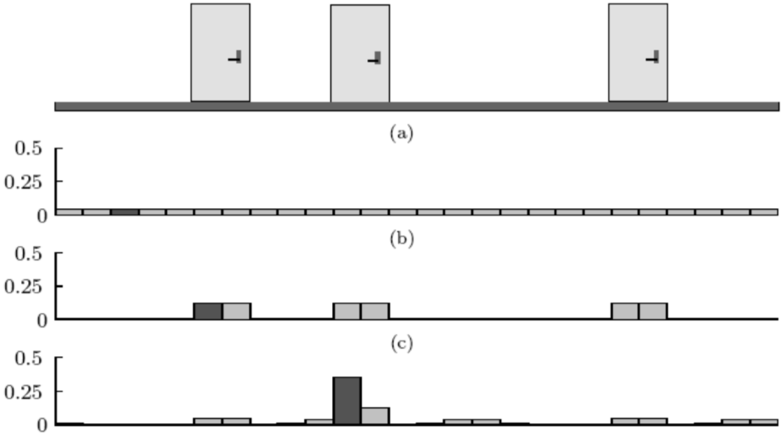
- Expanding $p(s'|a, b)$:

$$b'(s') = \frac{p(o|a, s') \sum_s p(s'|s, a) b(s)}{p(o|a, b)}$$

- Where:

$$p(o|a, b) = \sum_{s'} p(o|a, s') \sum_s p(s'|s, a) b(s)$$

Beliefs (4)



Most likely state

- Solve underlying MDP
- Condition actions on *most likely state*

$$\pi_{MLS} = \arg\max_a Q(s_{ML}, a)$$

where:

$$s_{ML} = \arg\max_s b(s)$$

Q_{MDP}

- Solve underlying MDP, select action with best expected value:

$$\pi_{QMDP} = \arg\max_a Q(b, a)$$

where:

$$Q(b, a) = \sum_s Q(s, a) b(s)$$

- Suppose $b(s_1) = 0.75$, $b(s_2) = 0.25$, and $Q(s, a)$ is:

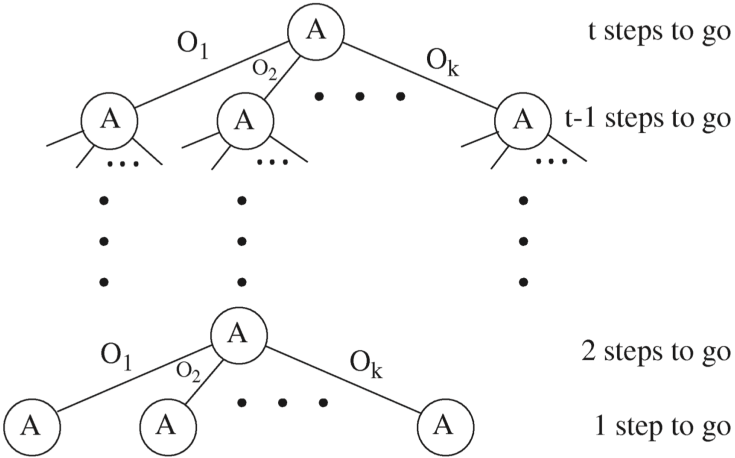
	s_1	s_2
a_1	100	100
a_2	101	0

- Will most likely state or Q_{MDP} yield a higher expected return?

Belief MDPs

- Belief is a *sufficient statistic* for history
- Therefore, we can define a *belief MDP*:
  - States are beliefs in the POMDP: $s_{BMDP} = b(s_{POMDP})$
  - Rewards are expectations wrt b: $R(b, a) = \sum_s b(s)R(s, a)$
  - Belief update happens in environment: $p(b'|b_t, a_t, o_t) = 1$ iff $b' = b_{t+1}$
- Automatically balance reward and information gathering
- Belief MDP has continuous state: belief vector has length $|S_{POMDP}|$
- Fortunately, the value function is *piecewise-linear and convex*
- What prevents *convenient delusions*?

Policy trees



POMDP value functions

- Value function of t-step policy tree π:

$$V^{\pi}(s) = R(s,a) + \gamma \sum_{s'} p(s'|s,a_{\pi}) \sum_{o} p(o|s',\pi_a) V^{\pi_o}(s')$$

  where π_o is the $(t-1)$-step policy subtree of π associated with o

- But we need value functions over beliefs, not states:

$$V^{\pi}(b) = \sum_{s \in S} b(s) V^{\pi}(s)$$

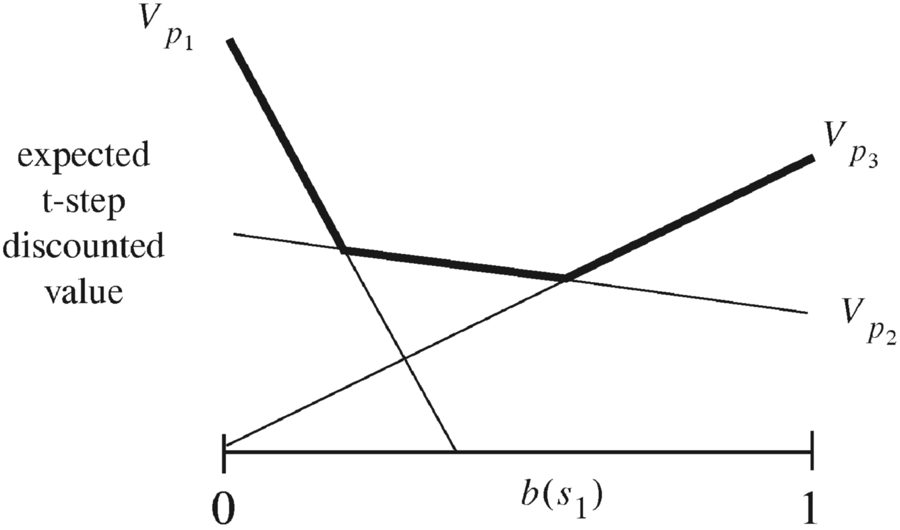
- For compactness, we write the state-value function as an *α-vector* $\alpha_{\pi} = \langle V_{\pi}(s_1), \ldots, V_{\pi}(s_{|S|}) \rangle$ such that:

$$V^{\pi}(b) = b \cdot \alpha_{\pi}$$

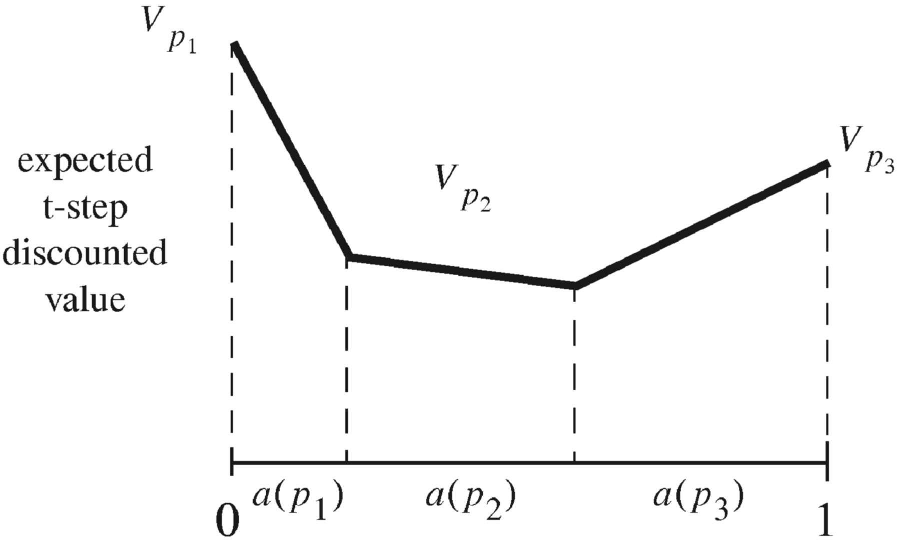
- The optimal value function is thus:

$$V^{*}(b) = \max_{\pi} b \cdot \alpha_{\pi}$$

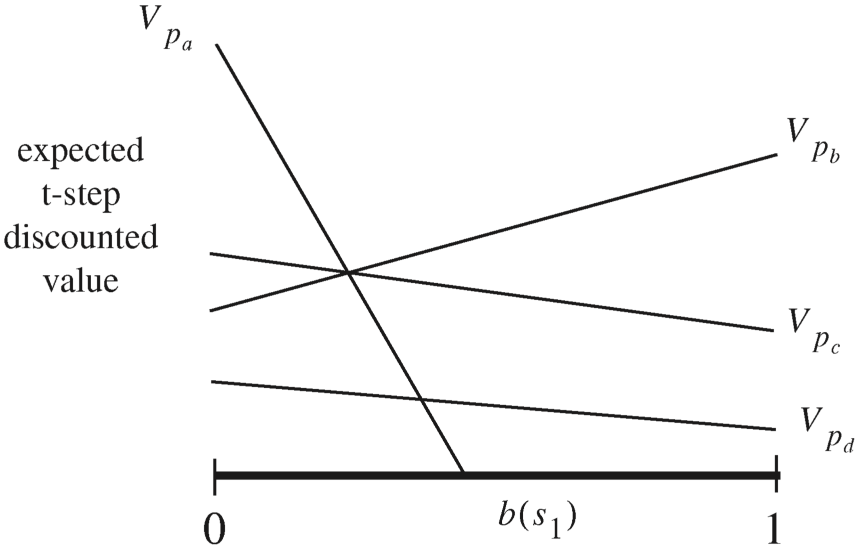
Piecewise-linear convex value functions



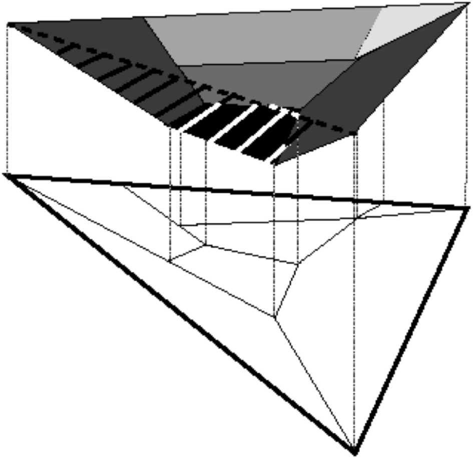
POMDP action selection



Dominated policy trees



POMDP value functions in 3D

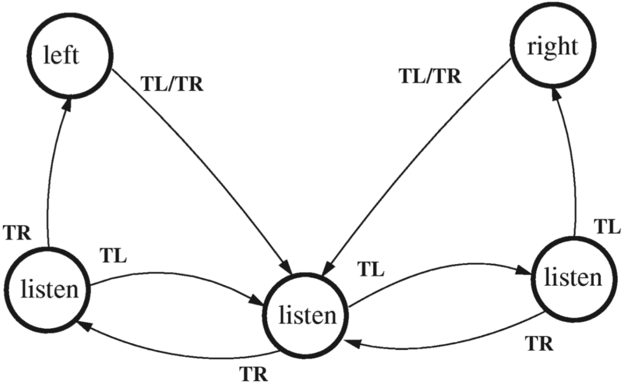


POMDP planning

- Value iteration [Sondik 1971, Monahan 1982]
  - Given set Π_{t-1} of undominated $(t-1)$-step policy trees
  - Construct all $|A||\Pi_{t-1}|^{|O|}$ t-step policy trees by extension
  - Prune dominated policy trees to form Π_t
- Faster: linear support [Chang 1988] & Witness [Cassandra et al. 1997]
- Approximate: point-based value iteration [Pineau et al. 2003]
- More scalable: on-line POMDP planning [Ross et al. 2008]

Infinite horizon planning

- Infinite horizon POMDP planning is undecidable!
- Optimal value function may have infinite facets
- Finite horizon planning may still converge for large t
- Yields finite state machine, e.g., infinite horizon tiger for $b_0 = 0.5, 0.5$:



Bayes-optimal reinforcement learning

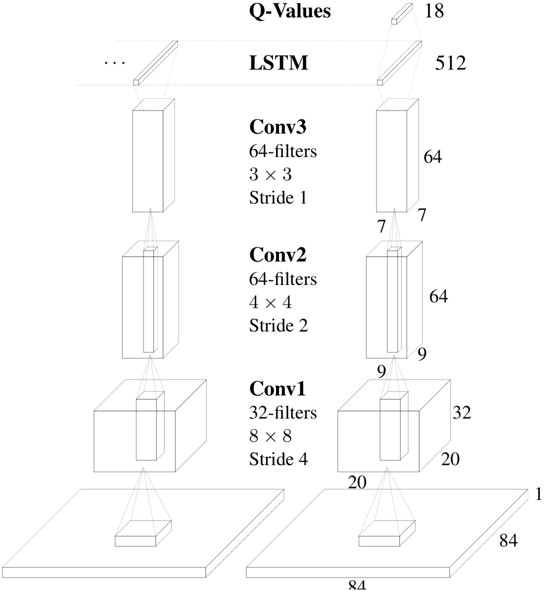
- Problem of learning in an MDP is cast as one of planning in a POMDP where the hidden state corresponds to the unknown model parameters: $s_{POMDP} = (s_{MDP}, T, R)$

- Like any other POMDP, this POMDP can be treated like a belief MDP: $s_{BMDP} = b(s_{POMDP})$

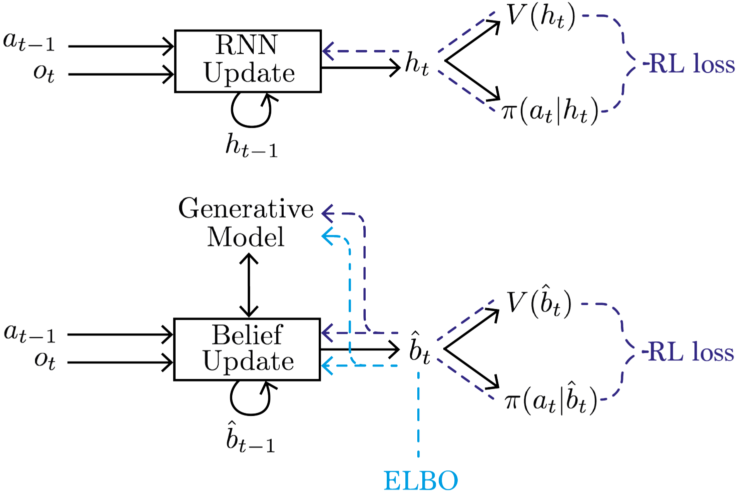
- However, since s_{MDP} is directly observed, only a belief over T and R is necessary, thus:

$$s_{BMDP} = b(s_{POMDP}) = b(s_{MDP}, T, R) = (s_{MDP}, b(T, R))$$

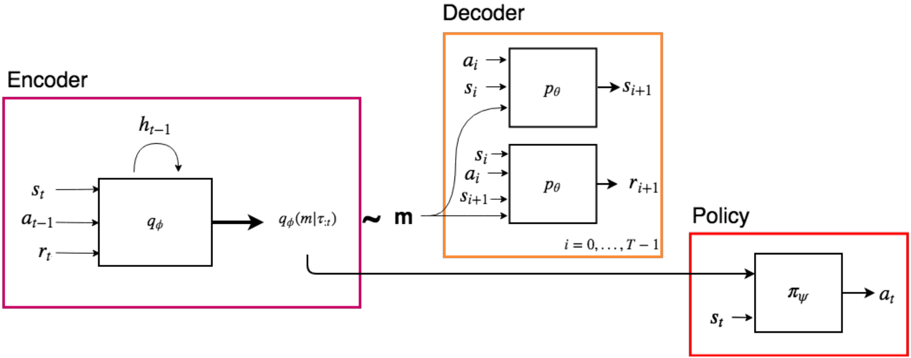
DRQN [Hausknecht & Stone 2015]



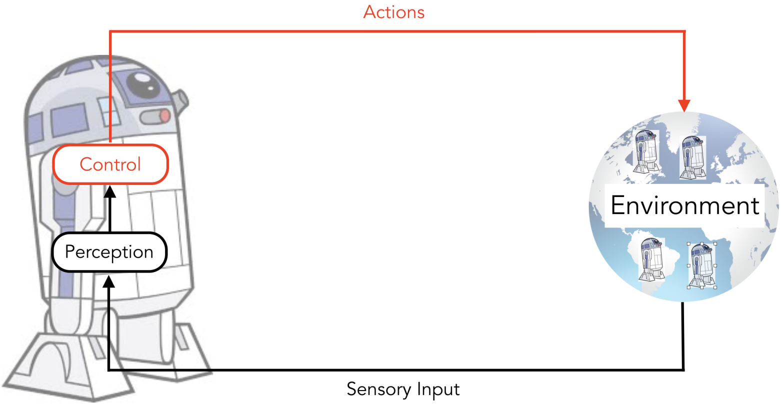
Deep Variational RL [Igl et al. 2018]



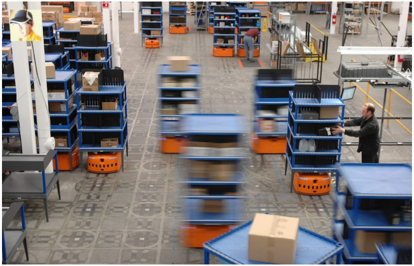
VariBAD [Zintgraf et al. 2019]



Multi-Agent Paradigm



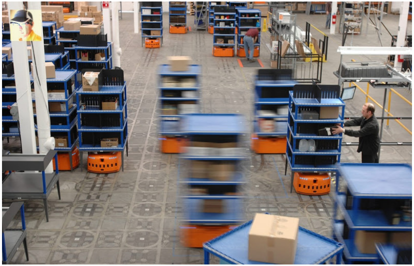
Multi-Agent Systems are Everywhere



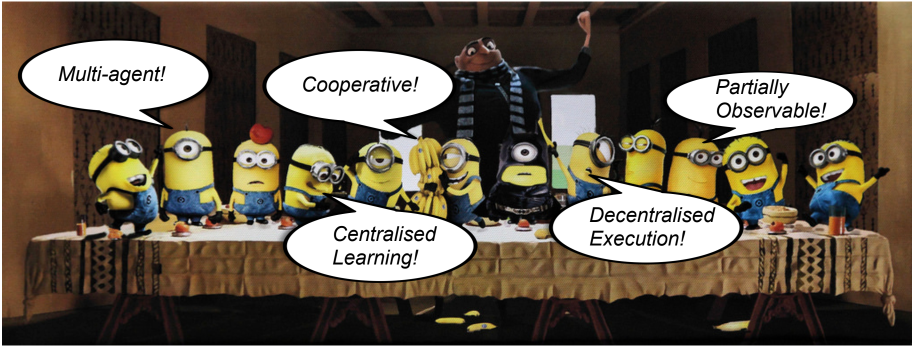
Types of Multi-Agent Systems

- *Cooperative:*
  - Shared team reward
  - Coordination problem
- *Competitive:*
  - Zero-sum games
  - Individual opposing rewards
  - Minimax equilibria
- *Mixed:*
  - General-sum games
  - Nash equilibria
  - What is the question? [Shoham et al. 2007]

Coordination Problems are Everywhere



Setting



(Figure by Jakob Foerster)

Multi-Agent MDP

- All agents see the global state s
- Individual actions: $u^a \in U$
- State transitions: $P(s'|s, \mathbf{u}) : S \times \mathbf{U} \times S \rightarrow [0, 1]$
- Shared team reward: $r(s, \mathbf{u}) : S \times \mathbf{U} \rightarrow \mathbb{R}$
- Equivalent to an MDP with a factored action space

Dec-POMDP

- Observation function: $O(s, a) : S \times A \to Z$
- Action-observation history: $\tau^a \in T \equiv (Z \times U)^*$
- Decentralised policies: $\pi^a(u^a|\tau^a) : T \times U \to [0, 1]$
- Natural decentralisation: communication and sensory constraints
- Artificial decentralisation: improve tractability
- Centralised learning of decentralised policies

The Predictability / Exploitation Dilemma

- Exploitation:
  - Maximising performance requires collecting reward
  - In a single-agent setting, this requires *exploiting* observations
- Predictability:
  - Dec-POMDP agents cannot explicitly communicate
  - Coordination requires *predictability*: "stick to the plan!"
  - Predictability can require ignoring private information

When does the benefit of exploiting private observations outweigh the cost in predictability?

Independent Learning

- Independent Q-learning [Tan 1993]
  - Each agent learns independently with its own Q-function
  - Treats other agents as part of the environment

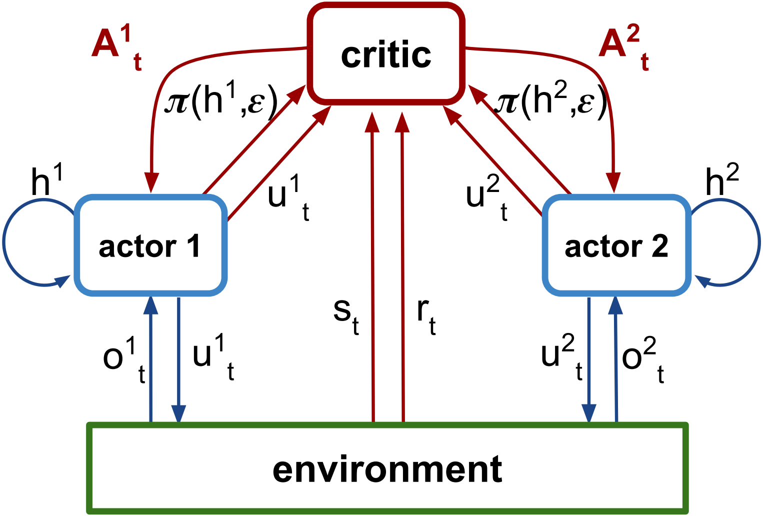
- Independent actor-critic [Foerster et al. 2018]
  - Each agent learns independently with its own actor-critic
  - Treats other agents as part of the environment

- Speed learning with *parameter sharing*
  - Different inputs, including a, induce different behaviour
  - Still independent: value functions condition only on τ^a and u^a

- Limitations:
  - Nonstationary learning
  - Hard to learn to coordinate

Centralised Critics [Lowe et al. 2017; Foerster et al. 2018]

Centralised $V(s, \boldsymbol{\tau})$ or $Q(s, \boldsymbol{\tau}, \mathbf{u})$ → hard greedification → actor-critic

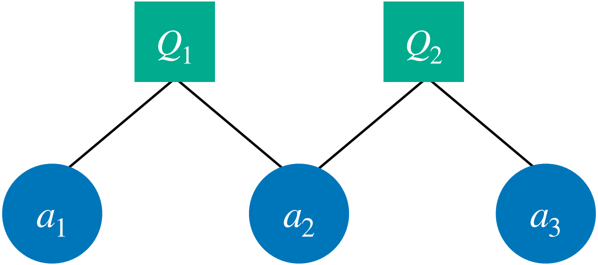


Factored Joint Value Functions

- *Factored value functions* [Guestrin et al. 2003] can improve scalability:

$$Q_{tot}(\boldsymbol{\tau}, \mathbf{u}; \boldsymbol{\theta}) = \sum_{e=1}^{E} Q_e(\tau^e, \mathbf{u}^e; \theta^e)$$

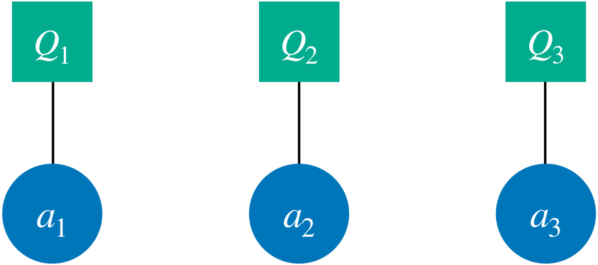
where each e indicates a subset of the agents



Value Decomposition Networks [Sunehag et al., 2017]

- Most extreme factorisation: one per agent:

$$Q_{tot}(\boldsymbol{\tau}, \mathbf{u}; \boldsymbol{\theta}) = \sum_{a=1}^{N} Q_a(\tau^a, u^a; \theta^a)$$



Decentralisability

- Added benefit of decentralising the max and arg max:

$$\max_{\mathbf{u}} Q_{tot}(\boldsymbol{\tau}, \mathbf{u}; \boldsymbol{\theta}) = \sum \max_{u^a} Q_a(\tau^a, u^a; \theta^a)$$

$$\arg\max_{\mathbf{u}} Q_{tot}(\boldsymbol{\tau}, \mathbf{u}; \boldsymbol{\theta}) = \begin{pmatrix} \arg\max_{u^1} Q_1(\tau^1, u^1; \theta^1) \\ \vdots \\ \arg\max_{u^n} Q_n(\tau^n, u^n; \theta^n) \end{pmatrix}$$

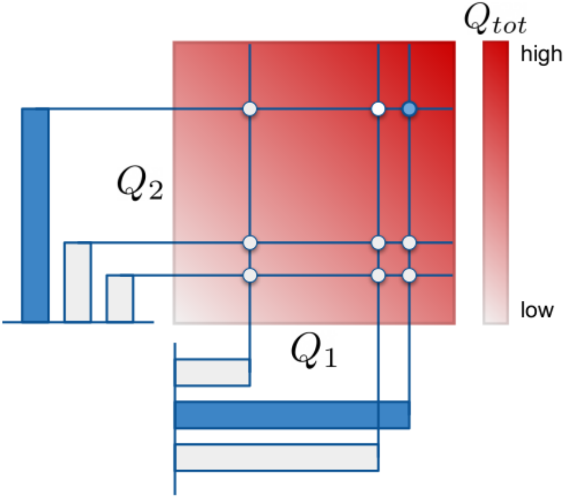
- No more hard greedification $\implies$ Q-learning, not actor-critic:

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{i=1}^{b} \left[\left(y_i^{\text{tot}} - Q_{tot}(\boldsymbol{\tau}, \mathbf{u}; \boldsymbol{\theta}) \right)^2 \right],$$

$$y_i^{\text{tot}} = r_i + \gamma \max_{\mathbf{u}'} Q_{tot}(\boldsymbol{\tau}'_i, \mathbf{u}'; \boldsymbol{\theta}^-)$$

QMIX's Monotonicity Constraint

To decentralise max / arg max, it suffices to enforce: $\frac{\partial Q_{tot}}{\partial Q_a} \geq 0, \ \forall a \in A$



Representational Capacity

It'll never work: monotonic mixing still can't capture the benefit of coordination

		Agent 2	
		A	*B*
Agent 1	*A*	0	1
	B	1	2

linear & monotonic

VDN & QMIX

		Agent 2	
		A	*B*
Agent 1	*A*	0	1
	B	1	8

nonlinear & monotonic

Just QMIX

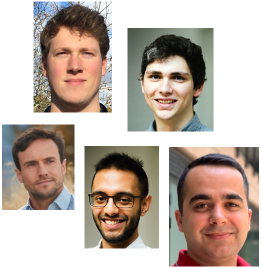
		Agent 2	
		A	*B*
Agent 1	*A*	2	1
	B	1	8

nonlinear & nonmonotonic

Neither

Bootstrapping

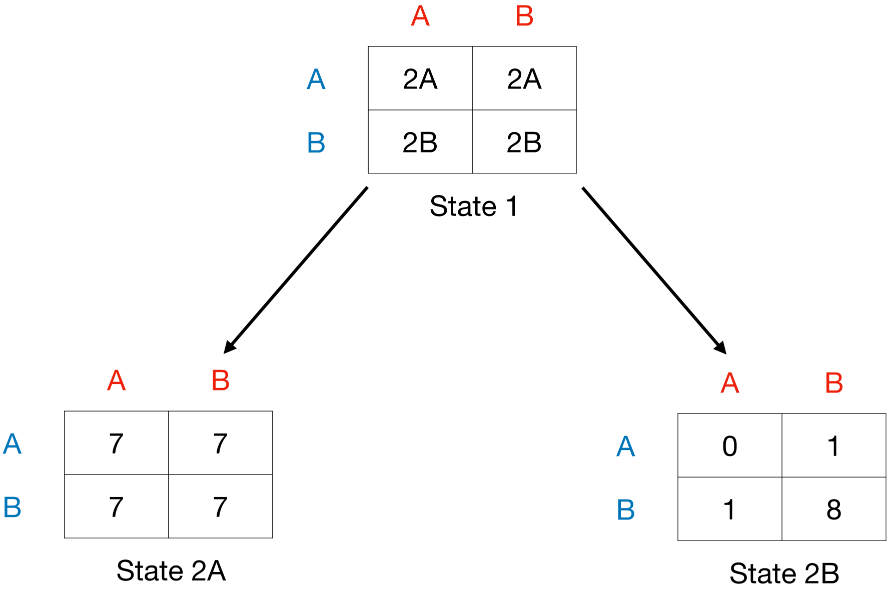
It matters because of *bootstrapping*



$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{i=1}^{b} \left[\left(y_i^{\text{tot}} - Q_{tot}(\boldsymbol{\tau}, \mathbf{u}, s; \boldsymbol{\theta}) \right)^2 \right],$$

$$y_i^{\text{tot}} = r_i + \gamma \max_{\mathbf{u}'} Q_{tot}(\boldsymbol{\tau}'_i, \mathbf{u}', s'; \boldsymbol{\theta}^-)$$

Two-Step Game



	A	B
A	2A	2A
B	2B	2B

State 1

	A	B
A	7	7
B	7	7

State 2A

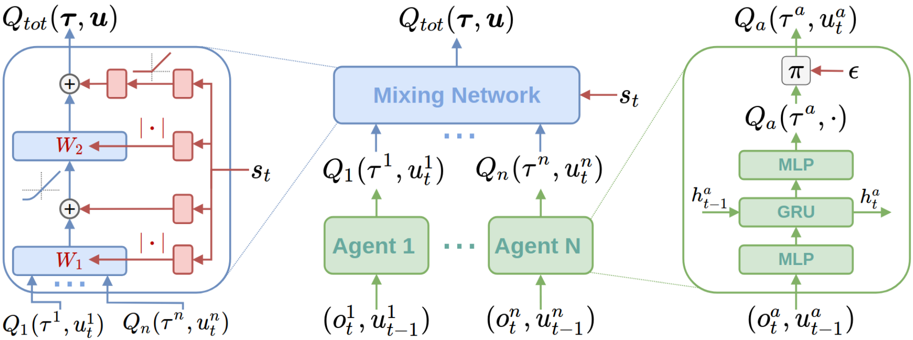
	A	B
A	0	1
B	1	8

State 2B

Two-Step Game Results

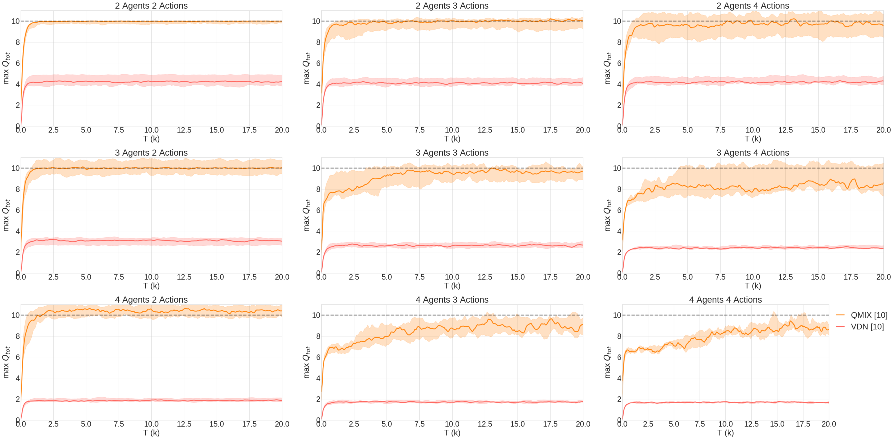
		A	B		A	B		A	B
Ground Truth	A	7	7	A	7	7	A	0	1
	B	8	8	B	7	7	B	1	8
		A	B		A	B		A	B
VDN	A	6.94	6.94	A	6.99	7.02	A	-1.87	2.31
	B	6.35	6.36	B	6.99	7.02	B	2.33	6.51
		A	B		A	B		A	B
QMIX	A	6.93	6.93	A	7.00	7.00	A	0.00	1.00
	B	7.92	7.92	B	7.00	7.00	B	1.00	8.00
		State 1			State 2A			State 2B	

QMIX [Rashid et al. 2018]



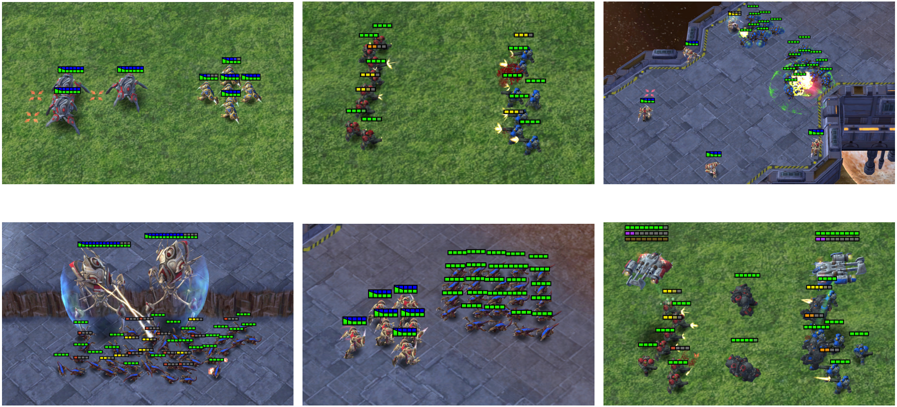
- Agent network: represents $Q_i(\tau^a, u^a; \theta^a)$
- Mixing network: represents $Q_{tot}(\boldsymbol{\tau})$ using nonnegative weights
- Hypernetwork: generates weights of hypernetwork based on global s

Random Matrix Games (The Students Were Right)



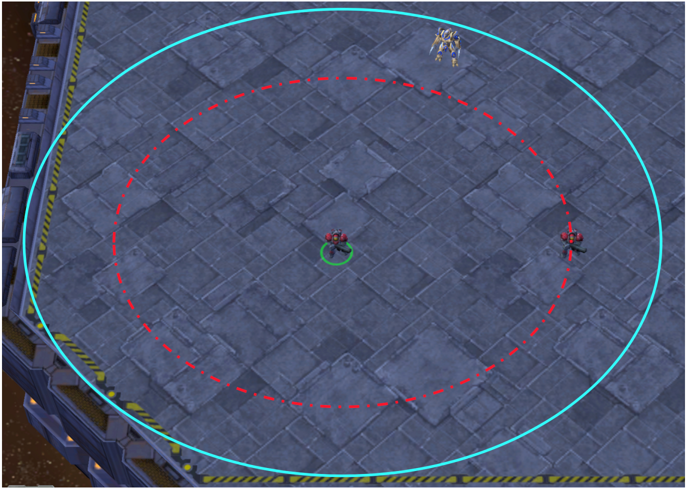
StarCraft Multi-Agent Challenge (SMAC)

[Samvelyan et al. 2019]



https://github.com/oxwhirl/smac
https://github.com/oxwhirl/pymarl

Partial Observability in SMAC

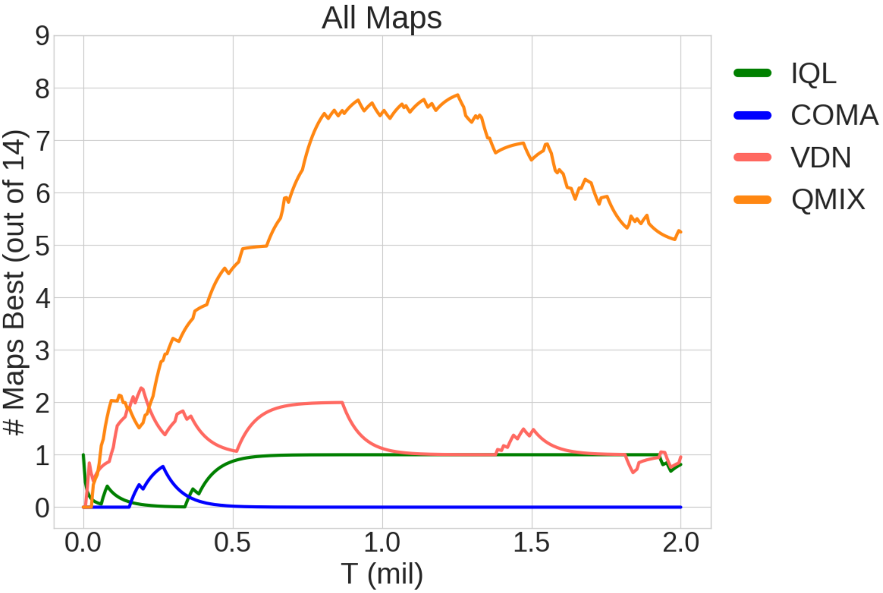


Cyan = sight range   Red = shooting range

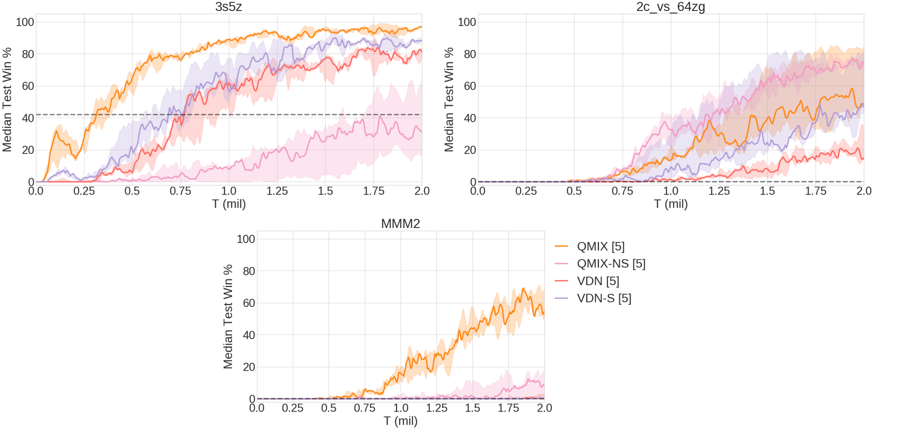
SMAC Maps

Name	Ally Units	Enemy Units
2s3z	2 Stalkers & 3 Zealots	2 Stalkers & 3 Zealots
3s5z	3 Stalkers & 5 Zealots	3 Stalkers & 5 Zealots
1c3s5z	1 Colossus, 3 Stalkers & 5 Zealots	1 Colossus, 3 Stalkers & 5 Zealots
5m_vs_6m	5 Marines	6 Marines
10m_vs_11m	10 Marines	11 Marines
27m_vs_30m	27 Marines	30 Marines
3s5z_vs_3s6z	3 Stalkers & 5 Zealots	3 Stalkers & 6 Zealots
MMM2	1 Medivac, 2 Marauders & 7 Marines	1 Medivac, 3 Marauders & 8 Marines
2s_vs_1sc	2 Stalkers	1 Spine Crawler
3s_vs_5z	3 Stalkers	5 Zealots
6h_vs_8z	6 Hydralisks	8 Zealots
bane_vs_bane	20 Zerglings & 4 Banelings	20 Zerglings & 4 Banelings
2c_vs_64zg	2 Colossi	64 Zerglings
corridor	6 Zealots	24 Zerglings

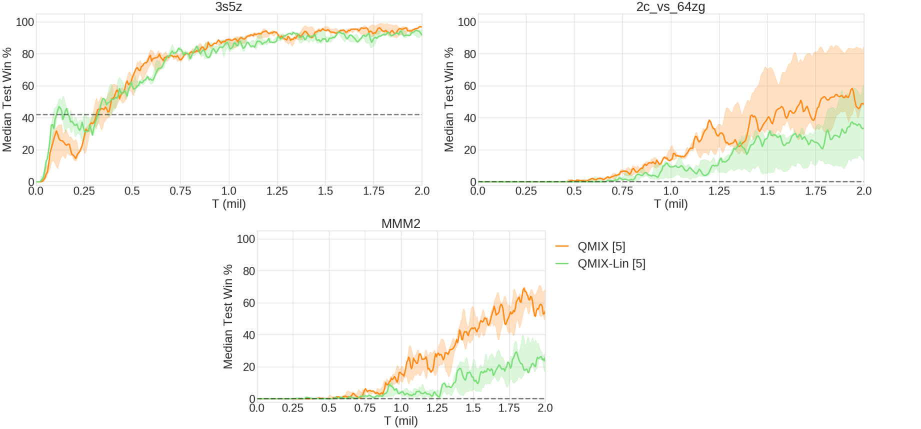
Overall Results (The Students Were Right)



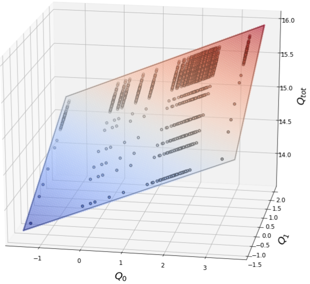
State Ablations



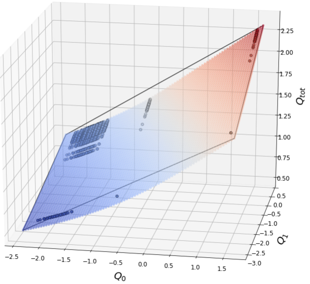
Linear Ablations



Learned Mixing Functions (2c_vs_64zg)

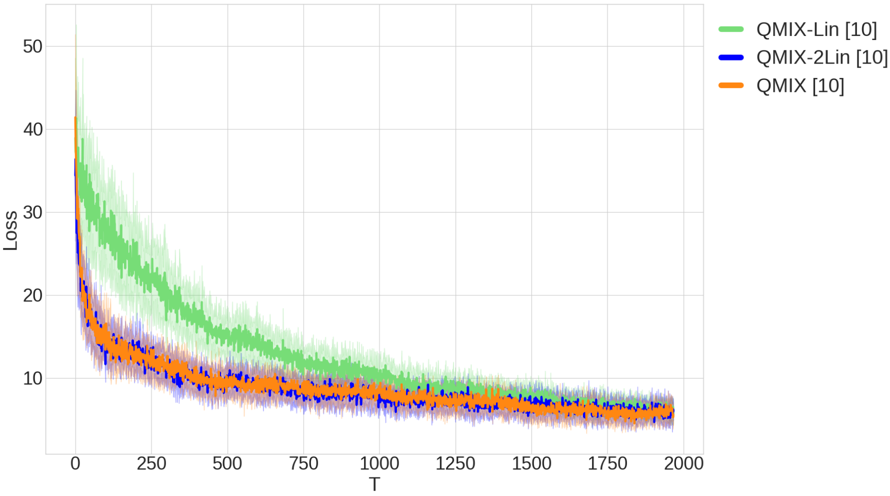


$t = 0$

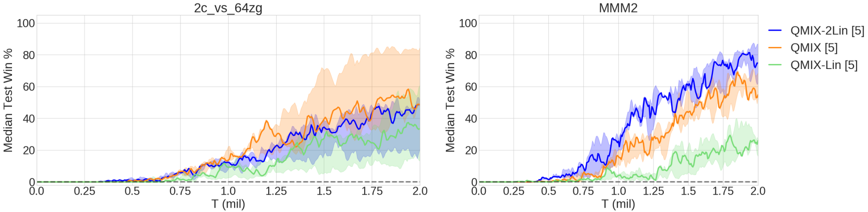


$t = 50$

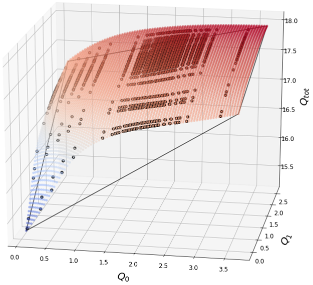
Multi-Layer Linear Mixing (Regression)



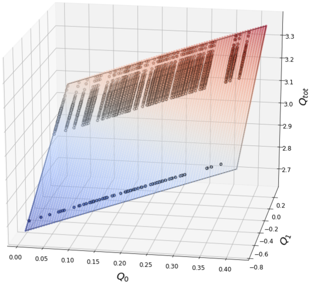
Multi-Layer Linear Mixing (SMAC)



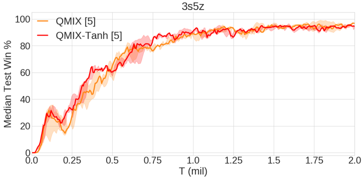
Tanh Activation

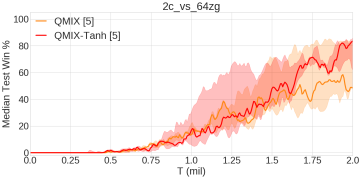


$t = 0$



$t = 50$





QMIX Takeaways

- Value function factorisation is crucial
- Flexible conditioning on central state is crucial
- Richly parameterised mixing is crucial
- Nonlinear mixing is not crucial (on SMAC)

Whiteson Research Lab

Using Reflexive Eye Movements for Fast Challenge-Response Authentication
A
#
Z
!
Q
&
F2
Ctrl
@
Shift
FIRE ACTION